

# *Decision tree algorithm*

## *short Weka tutorial*

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*Machine learning for Web Mining*  
a.a. 2009-2010

## Machine Learning: *brief summary*

### *Example*

You need to write a program that:

- given a Level Hierarchy of a company
- given an employe described trough some *attributes* (the number of attributes can be very high)
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### *Solution*

*Learn the function* to link each employe to the correct level.

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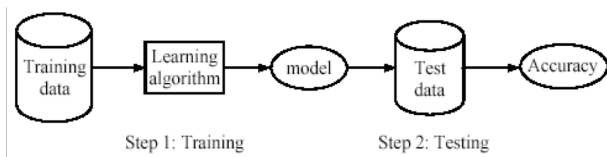
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# Learning Algorithms

- Probabilistic Functions (Bayesian Classifier)
- Functions to partitioning Vector Space
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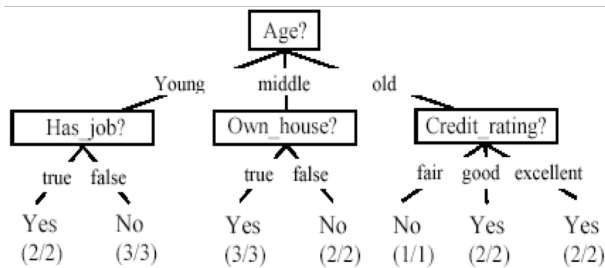
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- Boolean Functions (Decision Trees)

## Decision Tree: Domain Example

The class to learn is: approve a loan

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# Decision Tree



Decision Tree example for the loan problem

## *Is the decision tree unique?*

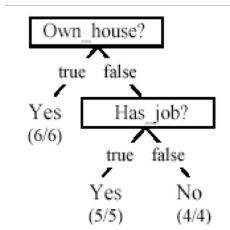
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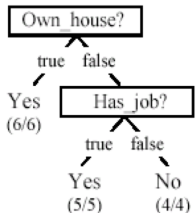
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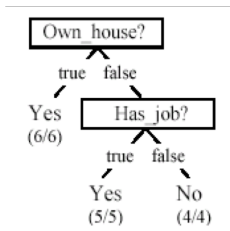


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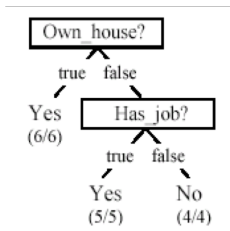
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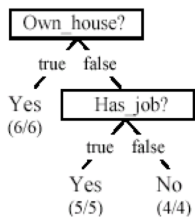
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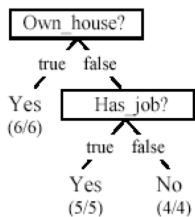


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- A decision tree can be converted to a set of rules .

# From a decision tree to a set of rules

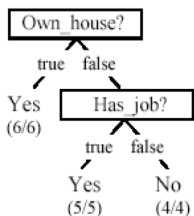


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Each path from the root to a leaf is a rule

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### Rules

Own\_house = true  $\rightarrow$  Class = yes

Own\_house = false , Has\_job = true  $\rightarrow$  Class = yes

Own\_house = false , Has\_job = false  $\rightarrow$  Class = no

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The heuristic is to choose the attribute with the maximum *Information Gain* or *Gain Ratio* based on information theory.

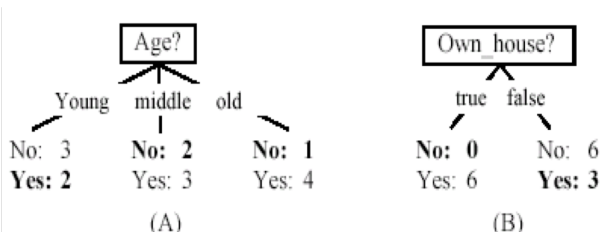
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# Information Gain

## Entropy of $D$

- Entropy is a measure of the uncertainty associated with a random variable.
- Given a set of examples  $D$  is possible to compute the original entropy of the dataset such as:

$$H[D] = - \sum_{j=1}^{|C|} P(c_j) \log_2 P(c_j)$$

where  $C$  is the set of desired class.

# Entropy

1. The data set  $D$  has 50% positive examples ( $\Pr(\text{positive}) = 0.5$ ) and 50% negative examples ( $\Pr(\text{negative}) = 0.5$ ).

$$\text{entropy}(D) = -0.5 \times \log_2 0.5 - 0.5 \times \log_2 0.5 = 1$$

2. The data set  $D$  has 20% positive examples ( $\Pr(\text{positive}) = 0.2$ ) and 80% negative examples ( $\Pr(\text{negative}) = 0.8$ ).

$$\text{entropy}(D) = -0.2 \times \log_2 0.2 - 0.8 \times \log_2 0.8 = 0.722$$

3. The data set  $D$  has 100% positive examples ( $\Pr(\text{positive}) = 1$ ) and no negative examples, ( $\Pr(\text{negative}) = 0$ ).

$$\text{entropy}(D) = -1 \times \log_2 1 - 0 \times \log_2 0 = 0$$

As the data become purer and purer, the entropy value becomes smaller and smaller.

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## Entropy of an attribute $A_i$

If we make attribute  $A_i$ , with  $v$  values, the root of the current tree, this will partition  $D$  into  $v$  subsets  $D_1, D_2, \dots, D_v$ . The expected entropy if  $A_i$  is used as the current root:

$$H_{A_i}[D] = \sum_{j=1}^v \frac{|D_j|}{|D|} H[D_j]$$

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Information gained by selecting attribute  $A_i$  to branch or to partition the data is given by the difference of *prior* entropy and the entropy of selected branch

$$\text{gain}(D, A_i) = H[D] - H_{A_i}[D]$$

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We choose the attribute with the *highest gain* to branch/split the current tree.

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## Back to the example

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## Example

9 examples belong to "YES" category  
and 6 to "NO". Exploiting prior  
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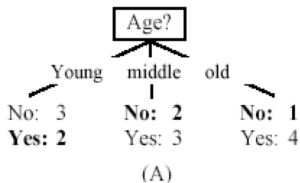
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while partitioning through the *Age* feature:

$$H_{Age}[D] = - \frac{5}{15} H[D_1] - \frac{5}{15} H[D_2] - \frac{5}{15} H[D_3] = 0.888$$

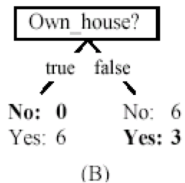
where

$$H[D_1] = - \frac{3}{3+2} \cdot \log_2 \left( \frac{3}{3+2} \right) - \frac{2}{3+2} \cdot \log_2 \left( \frac{2}{3+2} \right) = 0.971$$

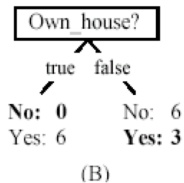
$$H[D_2] = - \frac{2}{2+3} \cdot \log_2 \left( \frac{2}{2+3} \right) - \frac{3}{2+3} \cdot \log_2 \left( \frac{3}{2+3} \right) = 0.971$$

$$H[D_3] = - \frac{1}{1+4} \cdot \log_2 \left( \frac{1}{1+4} \right) - \frac{4}{1+4} \cdot \log_2 \left( \frac{4}{1+4} \right) = 0.722$$

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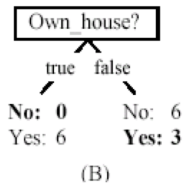


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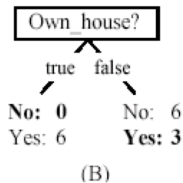
$$gain(D, Age) = 0.971 - 0.888 = 0.083$$

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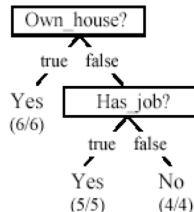
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- There are no examples left

# Algorithm for decision tree learning

```

1  Algorithm decisionTree( $D, A, T$ )
2  if  $D$  contains only training examples of the same class  $c_j \in C$  then
3      make  $T$  a leaf node labeled with class  $c_j$ ;
4  elseif  $A = \emptyset$  then
5      make  $T$  a leaf node labeled with  $c_j$ , which is the most frequent class in  $D$ 
6  else //  $D$  contains examples belonging to a mixture of classes. We select a single
7      // attribute to partition  $D$  into subsets so that each subset is purer
8       $p_0 = \text{impurityEval-1}(D)$ ;
9      for each attribute  $A_i \in \{A_1, A_2, \dots, A_k\}$  do
10          $p_i = \text{impurityEval-2}(A_i, D)$ 
11     end
12     Select  $A_g \in \{A_1, A_2, \dots, A_k\}$  that gives the biggest impurity reduction,
13     computed using  $p_0 - p_i$ ;
14     if  $p_0 - p_g < \text{threshold}$  then //  $A_g$  does not significantly reduce impurity  $p_0$ 
15         make  $T$  a leaf node labeled with  $c_j$ , the most frequent class in  $D$ .
16     else //  $A_g$  is able to reduce impurity  $p_0$ 
17         Make  $T$  a decision node on  $A_g$ ;
18         Let the possible values of  $A_g$  be  $v_1, v_2, \dots, v_m$ . Partition  $D$  into  $m$ 
19         disjoint subsets  $D_1, D_2, \dots, D_m$  based on the  $m$  values of  $A_g$ .
20         for each  $D_j$  in  $\{D_1, D_2, \dots, D_m\}$  do
21             if  $D_j \neq \emptyset$  then
22                 create a branch (edge) node  $T_j$  for  $v_j$  as a child node of  $T$ ;
23                 decisionTree( $D_j, A - \{A_g\}, T_j$ ) //  $A_g$  is removed
24             end
25         end
26     end
27 end

```

# *What is WEKA?*



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- Collection of ML algorithms - open-source Java package

**Site:**

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- For classification, Weka allows train/test split or Cross-fold validation
- Schemes for clustering:
  - EM and Cobweb

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{2 W, 4 "class B"}
```

- Note that the omitted values in a sparse instance are 0, they **are not** *missing* values! If a value is unknown, you must explicitly represent it with a question mark (?)

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- **Important generic options:**
  - `-t <training file>` **Specify training file**
  - `-T <test files>` **Specify Test file. If none testing is performed on training data**
  - `-x <number of folds>` **Number of folds for cross-validation**
  - `-l <input file>` **Use saved model**
  - `-d <output file>` **Output model to file**
  - `-split-percentage <train size>` **Size of training set**
  - `-c <class index>` **Index of attribute to use as class (NB: the index start from 1)**
  - `-p <attribute index>` **Only output the predictions and one attribute (0 for none) for all test instances.**